University of Essex
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Department of Government
TA: Stavros Poupakis

GV903: Advanced Research Methods

Class 8 Non-spherical Disturbances

Generate 1,000 observations of `x1, x2` drawn as $x_{i1} \sim N(0, 9)$ (note 9 is the variance!) and $x_{i2} \sim \chi^2_{(3)}$.

Then, generate an heteroskedastic error term using the following code

```
gen u = rnormal(0,4)
gen sigma = exp(0.3*x1)
gen e = sigma*u
```

That is $\epsilon_i \sim N(0, 16e^{0.6x_{i1}})$. Then, generate the dependent variable `y` to be

$$y_i = 10 + 5x_{i1} + 2x_{i2} + e_i$$

1. **Perform an OLS** regression of y on x_1 and x_2.

2. **Re-do the OLS using the Eicker-White heteroskedasticity-consistent estimator** of the variances. Type `help reg` to see how you can do that.

3. Since you know the diagonal matrix $\Omega = \text{diag}\{\sigma_i^2\}$ you can **calculate the GLS estimator**. Implement this in Stata using OLS on the transformed model

$$\frac{y_i}{\sqrt{\sigma_i^2}} = \frac{x_i'}{\sqrt{\sigma_i^2}}\beta + \frac{\epsilon_i}{\sqrt{\sigma_i^2}}$$

4. **Let's pretend now we don't know the DGP. We will use the Feasible GLS (FGLS)**. We need to use an approximation to σ_i^2, call it s_i^2 and estimate the following

$$\frac{y_i}{\sqrt{s_i^2}} = \frac{x_i'}{\sqrt{s_i^2}}\beta + \frac{\epsilon_i}{\sqrt{s_i^2}} \tag{1}$$

To calculate s_i^2 first you need to perform the following steps:

   a. Obtain the residuals e_i and the predicted values $\hat{y}_i$ from the OLS regression of y on x_1 and x_2 (with the heteroskedasticity-consistent Std. Errors).

   b. Regress $ln(e_i^2)$ on $\hat{y}_i$ and ${\hat{y}_i}^2$

   c. Calculate $s_i^2 = \hat{\alpha_0} + \hat{\alpha_1}\hat{y}_i + \hat{\alpha_2}{\hat{y}_i}^2$

   d. Use OLS in the transformed model in Equation (1).

5. **Put all the code together into a program**, which will return the coefficients for x_1 and x_2 from OLS, GLS and FGLS. **Perform a Monte Carlo simulation** to examine the properties of the estimators.